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GREEN'S FUNCTIONS FOR AN ANISOTROPIC MEDIUM: PART I. UNBOUNDED CASE

ARCON Corporation

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13. ABSTRACT (Maximum 200 words)

Dyadic Green's Function (DGF) for layered anisotropic media is essential for the electromagnetic field analysis of several problems. With the goal of deriving the DGF of a two-layer biaxially anisotropic medium we derive in this report the DGF of a corresponding unbounded problem. Using the Fourier transform method, an auxiliary dyadic Green's (ADGF) is first derived. The DGF is then obtained by performing a simple linear transformation on the ADGF. It is expressed in a compact dyadic form in terms of two characteristic waves, viz., the a-wave and the b-wave. Some features of the DGF are discussed by comparing our results with those of a corresponding uniaxial problem.

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1. INTRODUCTION

The need to study and understand electromagnetic wave propagation
and scattering in anisotropic media arises in such diverse areas
as optics, plasma physics, geophysics, antennas and remote sensing. In
some cases anisotropy occurs as an undesirable parasite; in some
others it is intentionally introduced to serve some special purpose.
Naturally occuring anisotropic materials which are of practical use
are very few (e.g., crystals). But thanks to the recent advances in
materials technology, a wide range of anisotropic materials with
diverse characteristics are readily available today. Alongside,
several novel applications of anisotropic materials are being
continuously invented [Kobayashi and Terakado, 1980; Paul and
Shevgaonkar, 1981][1,2].

One of the well-established tools for the analysis of electromagnetic
radiation and scattering problems is the method of Green's functions [Tai,
1971][3]. There have been a few investigations in the literature dealing
with the dyadic Green's function for anisotropic media. But they are
either restricted to the uniaxial case [Lee and Kong, 1983][4] or only
suitable to certain special applications [Krowne, 1984a and 1984b][5,6].
Krowne [1984b][6] has found his DGF to be very useful in problems such as
electromagnetic field analyses of integrated circuits on anisotropic
substrates. Although in the literature there are papers devoted to
electromagnetic scattering from anisotropic media of very general type
[Graglia and Uslenghi, 1984][7] we are particularly interested in analyses
which can offer some physical insight into the scattering mechanisms in
anisotropic media. With these in mind we seek to obtain the DGF of a two-

layer biaxially anisotropic medium whose principal axes can have any arbitrary orientations. There are several steps involved in our procedure. First we obtain the DGF for the unbounded problem. We next formulate the DGF for the two-layer problem, express the coefficients in terms of half-space Fresnel coefficients and finally evaluate the various half-space Fresnel coefficients. In this report we confine ourselves to the task of deriving the DGF for the unbounded case. Extension of this work to the two-layer problem will be treated in another report.

The contents of this report are organized as follows. Section 2 describes the formulation of the problem. In the next section we derive the DGF of the unbounded biaxial medium. In Section 4 we have a discussion of our result and the report concludes with a brief summary in Section 5.

2. FORMULATION OF THE PROBLEM

The geometry of the two-layer problem is shown in Figure 1. It consists of three regions - Region 0 ($z > 0$) is an isotropic medium with permittivity ϵ_0, Region 1 ($0 > z > -d$) is the anisotropic medium with permittivity $\overline{\epsilon}$ and Region 2 ($z < -d$) is an isotropic medium with permittivity ϵ_2. All three regions have the same permeability μ.

The permittivity of the medium in Region 1 is a second-rank tensor whose matrix representation in its principal coordinates, $\overline{\epsilon}^{(o)}$, take the following form.

$$\overline{\epsilon}^{(o)} = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \qquad (1)$$

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In order to obtain the representation of $\bar{\varepsilon}^{(o)}$ in the coordinate system of our problem (x, y, z) we perform two rotational transformations. Let x", y" and z" be the principal axes of the permittivity tensor. Note that the normal to the boundary is not along any of the principal axes in general. The first transformation (see Figure 2(a)) is an anticlockwise rotation through an angle ψ_1 about the x" axis. The second transformation (see Figure 2(b)) is an anticlockwise rotation through an angle ψ_2 about the z' axis. Thus the representation of the permittivity tensor in the (x, y, z) coordinate system is given by the following symmetric matrix.

$$\bar{\varepsilon} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33} \end{bmatrix} \tag{2}$$

where

$$\varepsilon_{11} = \varepsilon_x \cos^2\psi_2 + (\varepsilon_y \cos^2\psi_1 + \varepsilon_z \sin^2\psi_1) \sin^2\psi_2 \tag{3a}$$

$$\varepsilon_{12} = (- \varepsilon_x + \varepsilon_y \cos^2\psi_1 + \varepsilon_z \sin^2\psi_1) \sin\psi_2 \cos\psi_2 \tag{3b}$$

$$\varepsilon_{13} = (\varepsilon_z - \varepsilon_y) \sin\psi_1 \cos\psi_1 \sin\psi_2 \tag{3c}$$

$$\varepsilon_{22} = \varepsilon_x \sin^2\psi_2 + (\varepsilon_y \cos^2\psi_1 + \varepsilon_z \sin^2\psi_1) \cos^2\psi_2 \tag{3d}$$

$$\varepsilon_{23} = (\varepsilon_z - \varepsilon_y) \sin\psi_1 \cos\psi_1 \cos\psi_2 \tag{3e}$$

$$\varepsilon_{33} = \varepsilon_y \sin^2\psi_1 + \varepsilon_z \cos^2\psi_1 \tag{3f}$$

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We have an electric current source in Region 0 away from the boundary. We are interested in finding the DGF's : $\overline{\overline{G}}_{00}(\overline{r},\overline{r}')$, $\overline{\overline{G}}_{10}(\overline{r},\overline{r}')$ and $\overline{\overline{G}}_{20}(\overline{r},\overline{r}')$. The second subscript denotes the region where the source is located and the first subscript denotes the region containing the observation point. In order to obtain the above DGF's we need to solve the following equations [Tai, 1971][3]:

$$\nabla \times \nabla \times \overline{\overline{G}}_{00}(\overline{r},\overline{r}') - \omega^2 \mu \, \epsilon_0 \, \overline{\overline{G}}_{00}(\overline{r},\overline{r}') = \overline{\overline{I}} \, \delta(\overline{r} - \overline{r}') \qquad (4)$$

$$\nabla \times \nabla \times \overline{\overline{G}}_{10}(\overline{r},\overline{r}') - \omega^2 \mu \, \overline{\overline{\epsilon}} \, \overline{\overline{G}}_{10}(\overline{r},\overline{r}') = 0 \qquad (5)$$

$$\nabla \times \nabla \times \overline{\overline{G}}_{20}(\overline{r},\overline{r}') - \omega^2 \mu \, \epsilon_2 \, \overline{\overline{G}}_{20}(\overline{r},\overline{r}') = 0 \qquad (6)$$

where ω is the angular frequency. The boundary conditions associated with these DGF's are given as follows:

$$\hat{z} \times \overline{\overline{G}}_{00}(\overline{r},\overline{r}') = \hat{z} \times \overline{\overline{G}}_{10}(\overline{r},\overline{r}') \quad \text{at } z = 0 \qquad (7a)$$

$$\hat{z} \times \nabla \times \overline{\overline{G}}_{00}(\overline{r},\overline{r}') = \hat{z} \times \nabla \times \overline{\overline{G}}_{10}(\overline{r},\overline{r}') \quad \text{at } z = 0 \qquad (7b)$$

$$\hat{z} \times \overline{\overline{G}}_{10}(\overline{r},\overline{r}') = \hat{z} \times \overline{\overline{G}}_{20}(\overline{r},\overline{r}') \quad \text{at } z = -d \qquad (8a)$$

$$\hat{z} \times \nabla \times \overline{\overline{G}}_{10}(\overline{r},\overline{r}') = \hat{z} \times \nabla \times \overline{\overline{G}}_{20}(\overline{r},\overline{r}') \quad \text{at } z = -d \qquad (8b)$$

We find this boundary value problem too complicated to attack head on. Since most of the complications are due to the anisotropic medium we first obtain the DGF of the unbounded anisotropic medium. This is

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the problem we set ourselves to solve in this report.

3. DYADIC GREEN'S FUNCTION FOR THE UNBOUNDED PROBLEM

The DGF of the unbounded problem, $\bar{\bar{G}}(\bar{r},\bar{r}')$, satisfies the following equation:

$$\nabla \times \nabla \times \bar{\bar{G}}(\bar{r},\bar{r}') - \omega^2\mu \ \bar{\bar{\epsilon}} \ \bar{\bar{G}}(\bar{r},\bar{r}') = \bar{\bar{I}} \ \delta(\bar{r} - \bar{r}') \qquad (9)$$

Although this equation is very similar to (5), the major simplification lies in the absence of the complicated boundary conditions (7) and (8). The only condition to be satisfied here is the radiation condition.

Instead of seeking solution to (9) directly, we find it convenient to first introduce an 'auxiliary' dyadic Green's function (ADGF) $\bar{\bar{\mathcal{G}}}(\bar{r},\bar{r}')$ which is linearly related to DGF (see (36)). As we shall see the *dyadic decomposition is easier to achieve for ADGF which satisfies the* following equation:

$$\nabla \times \nabla \times \bar{\bar{\chi}} \ \bar{\bar{\mathcal{G}}}(\bar{r},\bar{r}') - \omega^2\mu \ \bar{\bar{\mathcal{G}}}(\bar{r},\bar{r}') = \bar{\bar{\epsilon}} \ \delta(\bar{r} - \bar{r}') \qquad (10)$$

where

$$\bar{\bar{\chi}} = \bar{\bar{\epsilon}}^{-1} \qquad (11)$$

Now we define the following Fourier transform pair:

$$\bar{\bar{\mathcal{G}}}(\bar{k},\bar{r}') = \int d^3\bar{r} \ \bar{\bar{\mathcal{G}}}(\bar{r},\bar{r}') \exp(-i\bar{k}\cdot\bar{r}) \qquad (12a)$$

$$\bar{\bar{\mathcal{G}}}(\bar{r},\bar{r}') = \frac{1}{(2\pi)^3} \int d^3\bar{k} \ \bar{\bar{\mathcal{G}}}(\bar{k},\bar{r}') \exp(i\bar{k}\cdot\bar{r}) \qquad (12b)$$

Then in the Fourier domain, (10) reduces to the following algebraic

equation.

$$\bar{k} \times \bar{k} \times \bar{\bar{\chi}} \, \bar{\mathfrak{I}}(\bar{k},\bar{r}') + \omega^2 \mu \, \bar{\mathfrak{I}}(\bar{k},\bar{r}') = - \bar{\bar{\epsilon}} \exp(-i\bar{k}\cdot\bar{r}') \qquad (13)$$

The solution to (13) is readily obtained as

$$\bar{\mathfrak{I}}(\bar{k},\bar{r}') = - \left[\bar{k}^2 \, \bar{\bar{\chi}} + \omega^2 \mu \, \bar{\bar{I}} \right]^{-1} \bar{\bar{\epsilon}} \exp(-i\,\bar{k}\cdot\bar{r}') \qquad (14)$$

where the matrix representation of $\bar{k}$ in the (x, y, z) coordinate system is given as

$$\bar{k} = \begin{bmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{bmatrix} \qquad (15)$$

Substituting (14) in (12b), we get

$$\bar{\mathfrak{I}}(\bar{r},\bar{r}') = - \frac{1}{(2\pi)^3} \int d^3\bar{k} \left\{ \frac{\mathrm{adj}\left(\bar{k}^2 \, \bar{\bar{\chi}} + \omega^2 \mu \, \bar{\bar{I}} \right)}{\det\left[\bar{k}^2 \, \bar{\bar{\chi}} + \omega^2 \mu \, \bar{\bar{I}} \right]} \, \bar{\bar{\epsilon}} \exp\left[i\bar{k}\cdot(\bar{r}-\bar{r}') \right] \right\} \qquad (16)$$

Factorizing $\det\left[\bar{k}^2 \, \bar{\bar{\chi}} + \omega^2 \mu \, \bar{\bar{I}} \right]$, we obtain

$$\det\left[\bar{k}^2 \, \bar{\bar{\chi}} + \omega^2 \mu \, \bar{\bar{I}} \right] = \omega^2 \mu \, \epsilon_{33} \det \bar{\bar{\chi}} \left(k_z - k_z^{au} \right)$$

$$\cdot \left(k_z - k_z^{ad} \right) \left(k_z - k_z^{bu} \right) \left(k_z - k_z^{bd} \right) \qquad (17)$$

Here

$$\left\{ \begin{matrix} k_z^{au} \\ k_z^{ad} \end{matrix} \right\} = \frac{1}{2} \left(- \mathcal{A}/2 \pm \mathcal{R} \pm \mathcal{D} \right) \qquad (18a)$$

$$\left\{ \begin{matrix} k_z^{bu} \\ k_z^{bd} \end{matrix} \right\} = \frac{1}{2} \left(-\mathcal{A}/2 \pm \mathcal{R} \pm \mathcal{E} \right) \tag{18b}$$

where $\mathcal{A}$, $\mathcal{R}$, $\mathcal{D}$ and $\mathcal{E}$ are given in the Appendix. The superscripts a and b refer to the two characteristic waves that exist in a biaxially anisotropic medium. Hereafter we shall denote them as a-wave and b-wave. The superscripts u and d denote the upward and downward propagating waves, respectively. Note that Re k_z^{au} and Re k_z^{bu} are positive while Re k_z^{ad} and Re k_z^{bd} are negative. These are in agreement with our convention for upward and downward travelling waves. Furthermore, we assume that the medium is slightly lossy so that Re $k_z^{\zeta i} \gg$ Im $k_z^{\zeta i}$, $\zeta =$ a or b and i = u or d. Hereafter in this paper we follow the above notation for ζ and i.

From (16) and (17) we obtain

$$\overline{\mathfrak{J}}(\overline{r},\overline{r}') = -\frac{1}{(2\pi)^3} \frac{\varepsilon_x \varepsilon_y \varepsilon_z}{\omega^2 \mu \ \varepsilon_{33}} \int_{-\infty}^{\infty} dk_z \int d^2\overline{k}_\rho \ e^{i\overline{k}_\rho \cdot (\overline{\rho}-\overline{\rho}')} \ e^{ik_z(z-z')}$$

$$\cdot \ \frac{\text{adj} \left(\overline{k}^2 \ \overline{\overline{\chi}} + \omega^2\mu \ \overline{\overline{I}} \right) \overline{\overline{\varepsilon}}}{(k_z - k_z^{au})(k_z - k_z^{ad})(k_z - k_z^{bu})(k_z - k_z^{bd})} \tag{19}$$

We now perform the integration in (19) over k_z. The integration is from $-\infty$ to $+\infty$ along the real axis. There are four poles in the complex k_z - plane, viz., at k_z^{au}, k_z^{ad}, k_z^{bu} and k_z^{bd}. The poles at k_z^{au} and k_z^{bu} lie in the first quadrant while the poles at k_z^{ad} and k_z^{bd} lie in the third quadrant. For z > z' we deform the integration contour in the upper half-plane and this contour encloses the poles at k_z^{au} and k_z^{bu}.

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Using Cauchy's theorem the following result is readily obtained.
For $z > z'$

$$\bar{\bar{\mathfrak{G}}}(\bar{r},\bar{r}') = -\frac{i}{(2\pi)^2} \frac{\epsilon_x \epsilon_y \epsilon_z}{\omega^2 \mu \, \epsilon_{33}} \int d^2\bar{k}_\rho$$

$$\cdot \left\{ \frac{\lambda^{au} g^{au} \, \hat{a}^+ \, \hat{a}^+}{(k_z^{au}-k_z^{ad})(k_z^{au}-k_z^{bu})(k_z^{au}-k_z^{bd})} \; \exp\left[i\bar{k}^a \cdot (\bar{r}-\bar{r}')\right] \right.$$

$$\left. + \frac{\lambda^{bu} g^{bu} \, \hat{b}^+ \, \hat{b}^+}{(k_z^{bu}-k_z^{au})(k_z^{bu}-k_z^{ad})(k_z^{bu}-k_z^{bd})} \; \exp\left[i\bar{k}^b \cdot (\bar{r}-\bar{r}')\right] \right\}$$

$$(20)$$

where

$$\hat{a}^+ = \frac{1}{h^{au}} \left[\frac{\hat{k}^a \times \hat{o}_1}{|\hat{k}^a \times \hat{o}_1|} + \frac{\hat{k}^a \times \hat{o}_2}{|\hat{k}^a \times \hat{o}_2|} \right] \qquad (21a)$$

$$\hat{b}^+ = \frac{1}{h^{bu}} \left[\frac{\hat{k}^b \times (\hat{k}^b \times \hat{o}_1)}{|\hat{k}^b \times \hat{o}_1|} + \frac{\hat{k}^b \times (\hat{k}^b \times \hat{o}_2)}{|\hat{k}^b \times \hat{o}_2|} \right] \qquad (21b)$$

$$\bar{k}^\varsigma = \hat{\rho} \, k_\rho + \hat{z} \, k_z^{\varsigma u} \quad , \quad \varsigma = a \text{ or } b \qquad (22)$$

$$\lambda^{\varsigma u} = \frac{(k^\varsigma)^2}{\omega^2 \mu - (k^\varsigma)^2 \, \chi_1^{\varsigma u}} \qquad (23)$$

$$g^{\varsigma u} = (k^\varsigma)^4 \, (\chi_2^{\varsigma u})^2 + \left[\omega^2 \mu - (k^\varsigma)^2 \, \chi_1^{\varsigma u} \right]^2 \qquad (24)$$

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$$h^{\zeta u} = \sqrt{2} \left\{ 1 + \frac{(\hat{k}^{\zeta} \times \hat{o}_1) \cdot (\hat{k}^{\zeta} \times \hat{o}_2)}{|\hat{k}^{\zeta} \times \hat{o}_1| \quad |\hat{k}^{\zeta} \times \hat{o}_2|} \right\}^{1/2} \qquad (25)$$

$$\hat{k}^{\zeta} = \overline{k}^{\zeta} / k^{\zeta} \qquad (26a)$$

$$k^{\zeta} = |\overline{k}^{\zeta}| \qquad (26b)$$

$$\left\{ \begin{array}{c} \hat{o}_1 \\ \hat{o}_2 \end{array} \right\} = \hat{x} \left(\pm g_1 \cos \psi_2 + g_2 \sin \psi_1 \sin \psi_2 \right)$$

$$+ \hat{y} \left(\mp g_1 \sin \psi_2 + g_2 \sin \psi_1 \cos \psi_2 \right) + \hat{z} \cos \psi_1 \qquad (27)$$

$$g_1 = \left[\frac{\epsilon_z(\epsilon_y - \epsilon_x)}{\epsilon_y(\epsilon_z - \epsilon_x)} \right]^{1/2} \qquad (28a)$$

$$g_2 = \left[\frac{\epsilon_x(\epsilon_z - \epsilon_y)}{\epsilon_y(\epsilon_z - \epsilon_x)} \right]^{1/2} \qquad (28b)$$

$$x_1^{\zeta u} = x_{11} \sin^2 \phi^{\zeta u} + x_{22} \cos^2 \phi^{\zeta u} - 2 x_{12} \sin \phi^{\zeta u} \cos \phi^{\zeta u} \qquad (29a)$$

$$x_2^{\zeta u} = \left(x_{11} - x_{22} \right) \cos \theta^{\zeta u} \sin \phi^{\zeta u} \cos \phi^{\zeta u}$$

$$+ x_{12} \cos \theta^{\zeta u} \left(\sin^2 \phi^{\zeta u} - \cos^2 \phi^{\zeta u} \right)$$

$$+ \left(x_{23} \cos \phi^{\zeta u} - x_{13} \sin \phi^{\zeta u} \right) \sin \theta^{\zeta u} \qquad (29b)$$

χ_{ij} is the ij th element of the matrix $\overline{\overline{\chi}}$; the angles $\left(\theta^{\varsigma u}, \phi^{\varsigma u}\right)$ denote the direction of $\hat{k}^{\varsigma}$. For definiteness we have assumed in the above equations that $\epsilon_x < \epsilon_y < \epsilon_z$.

For $z < z'$ we deform the integration contour of (19) in the lower half-plane thus enclosing the poles at k_z^{ad} and k_z^{bd}. Once again, use of Cauchy's theorem leads to the following result.

For $z < z'$

$$\overline{\overline{\mathfrak{G}}}(\overline{r},\overline{r}') = \frac{i}{(2\pi)^2} \frac{\epsilon_x \epsilon_y \epsilon_z}{\omega^2 \mu \, \epsilon_{33}} \int d^2\overline{k}_\rho$$

$$\cdot \left\{ \frac{\lambda^{ad} \, g^{ad} \, \hat{a}^- \, \hat{a}^-}{(k_z^{ad} - k_z^{au})(k_z^{ad} - k_z^{bu})(k_z^{ad} - k_z^{bd})} \exp\left[i\overline{\kappa}^a \cdot (\overline{r} - \overline{r}')\right] \right.$$

$$\left. + \frac{\lambda^{bd} \, g^{bd} \, \hat{b}^- \, \hat{b}^-}{(k_z^{bd} - k_z^{au})(k_z^{bd} - k_z^{ad})(k_z^{bd} - k_z^{bu})} \exp\left[i\overline{\kappa}^b \cdot (\overline{r} - \overline{r}')\right] \right\}$$

$$\tag{30}$$

where

$$\hat{a}^- = \frac{1}{h^{ad}} \left[\frac{\hat{\kappa}^a \times \hat{o}_1}{|\hat{\kappa}^a \times \hat{o}_1|} + \frac{\hat{\kappa}^a \times \hat{o}_2}{|\hat{\kappa}^a \times \hat{o}_2|} \right] \tag{31a}$$

$$\hat{b}^- = \frac{1}{h^{bd}} \left[\frac{\hat{\kappa}^b \times (\hat{\kappa}^b \times \hat{o}_1)}{|\hat{\kappa}^b \times \hat{o}_1|} + \frac{\hat{\kappa}^b \times (\hat{\kappa}^b \times \hat{o}_2)}{|\hat{\kappa}^b \times \hat{o}_2|} \right] \tag{31b}$$

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$$\vec{\kappa}^\varsigma = \hat{\rho}\, k_\rho + \hat{z}\, k_z^{\varsigma d} \quad , \quad \varsigma = a \text{ or } b \tag{32a}$$

$$\hat{\kappa}^\varsigma = \vec{\kappa}^\varsigma / |\vec{\kappa}^\varsigma| \tag{32b}$$

$$\lambda^{\varsigma d} = \lambda^{\varsigma u} \left\{ u \to d \right\} \tag{33}$$

$$g^{\varsigma d} = g^{\varsigma u} \left\{ u \to d \right\} \tag{34}$$

$$h^{\varsigma d} = h^{\varsigma u} \left\{ u \to d \right\} \tag{35}$$

Now that we have derived the ADGF $\overline{\overline{\mathcal{G}}}(\overline{r},\overline{r}')$, it is straightforward to obtain the DGF $\overline{\overline{G}}(\overline{r},\overline{r}')$. On post-multiplying (10) by $\overline{\chi}$ and comparing it with (9), we deduce that the DGF is linearly related to the ADGF as

$$\overline{\overline{G}}(\overline{r},\overline{r}') = \overline{\chi}\, \overline{\overline{\mathcal{G}}}(\overline{r},\overline{r}')\, \overline{\chi} \tag{36}$$

Thus the DGF for the unbounded problem is finally written as follows.
For $z > z'$

$$\overline{\overline{G}}(\overline{r},\overline{r}') = -\frac{i}{(2\pi)^2}\ \frac{\epsilon_x \epsilon_y \epsilon_z}{\omega^2 \mu\, \epsilon_{33}}\ \int d^2\overline{k}_\rho$$

$$\cdot \left\{ \frac{\nu^{au}\, \lambda^{au}\, g^{au}\, \hat{a}^+\, \hat{a}^+}{(k_z^{au} - k_z^{ad})(k_z^{au} - k_z^{bu})(k_z^{au} - k_z^{bd})}\ \exp\left[i\overline{k}^a \cdot (\overline{r} - \overline{r}') \right] \right.$$

$$\left. + \frac{\nu^{bu}\, \lambda^{bu}\, g^{bu}\, \hat{b}^+\, \hat{b}^+}{(k_z^{bu} - k_z^{au})(k_z^{bu} - k_z^{ad})(k_z^{bu} - k_z^{bd})}\ \exp\left[i\overline{k}^b \cdot (\overline{r} - \overline{r}') \right] \right\}$$

$$\tag{37}$$

and for $z < z'$

$$\overline{G}(\overline{r},\overline{r}') = \frac{i}{(2\pi)^2} \frac{\epsilon_x \epsilon_y \epsilon_z}{\omega^2 \mu \epsilon_{33}} \int d^2\overline{k}_\rho$$

$$\cdot \left\{ \frac{\nu^{ad} \lambda^{ad} g^{ad} \hat{a}^- \hat{a}^-}{(k_z^{ad}-k_z^{au})(k_z^{ad}-k_z^{bu})(k_z^{ad}-k_z^{bd})} \exp\left[i\overline{\kappa}^a \cdot (\overline{r}-\overline{r}')\right] \right.$$

$$\left. + \frac{\nu^{bd} \lambda^{bd} g^{bd} \hat{b}^- \hat{b}^-}{(k_z^{bd}-k_z^{au})(k_z^{bd}-k_z^{ad})(k_z^{bd}-k_z^{bu})} \exp\left[i\overline{\kappa}^b \cdot (\overline{r}-\overline{r}')\right] \right\}$$

$$(38)$$

where

$$\hat{a}^+ = \left(\nu^{au}\right)^{-1/2} \overline{\overline{\chi}} \cdot \hat{a}^+ \tag{39a}$$

$$\hat{a}^- = \left(\nu^{ad}\right)^{-1/2} \overline{\overline{\chi}} \cdot \hat{a}^- \tag{39b}$$

$$\hat{b}^+ = \left(\nu^{bu}\right)^{-1/2} \overline{\overline{\chi}} \cdot \hat{b}^+ \tag{40a}$$

$$\hat{b}^- = \left(\nu^{bd}\right)^{-1/2} \overline{\overline{\chi}} \cdot \hat{b}^- \tag{40b}$$

$$\nu^{\ell u} = \hat{\ell}^+ \cdot \overline{\overline{\chi}}^2 \cdot \hat{\ell}^+ , \qquad \ell = a \text{ or } b \tag{41a}$$

$$\nu^{\ell d} = \hat{\ell}^- \cdot \overline{\overline{\chi}}^2 \cdot \hat{\ell}^- , \qquad \ell = a \text{ or } b \tag{41b}$$

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4. DISCUSSION

Having now derived the DGF of the unbounded biaxially anisotropic medium, it will be instructive to study some of its characteristics. First we compare our DGF with those of a corresponding uniaxial problem [Lee and Kong, 1983][4]. We notice that our result is identical in structure to theirs. The most important difference is that in the place of ordinary and extraordinary waves we have a- and b- waves.

In order to study the characteristics of a- and b- waves we first examine the dispersion equation

$$\det \left[\ \bar{k}^2 \ \bar{\bar{\chi}} + \omega^2 \mu \ \bar{\bar{I}} \ \right] \ = 0 \tag{42}$$

When we solve (42) for k_z we have found in (18) that there are four solutions. However, when we solve (42) for k^2 we find that it has two solutions, viz., $(k^a)^2$ and $(k^b)^2$, where

$$(k^a)^2 \ = \frac{\omega^2 \mu}{2 \ \tilde{d}_a} \left[\ t_a + \left(\ t_a^2 - 4 \ \tilde{d}_a \right)^{1/2} \ \right] \tag{43a}$$

$$(k^b)^2 \ = \frac{\omega^2 \mu}{2 \ \tilde{d}_b} \left[\ t_b - \left(\ t_b^2 - 4 \ \tilde{d}_b \right)^{1/2} \ \right] \tag{43b}$$

$$t_\varsigma \ = \text{tr} \ \bar{\bar{\chi}} - \hat{k}^\varsigma \cdot \bar{\bar{\chi}} \cdot \hat{k}^\varsigma \tag{44a}$$

$$\tilde{d}_\varsigma \ = \left(\ \epsilon_x \ \epsilon_y \ \epsilon_z \ \right) \hat{k}^\varsigma \cdot \bar{\bar{\epsilon}} \cdot \hat{k}^\varsigma \tag{44b}$$

It is clear from (43) that the propagation constants k^a and k^b are

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functions of the direction of propagation. This implies that both a-
and b- waves belong to the extraordinary category.

But when we let $\epsilon_x = \epsilon_y$ and $\psi_2 = 0°$, (43) becomes

$$(k^a)^2 = \omega^2 \mu \, \epsilon_x \tag{45a}$$

$$(k^b)^2 = \omega^2 \mu \, \epsilon_z + (k_z^{bu})^2 \left(1 - \epsilon_z/\epsilon_x \right) \tag{45b}$$

We now see that k^b is dependent on the direction of propagation while k^a is not. Thus in this special situation the a-wave is an ordinary wave and the b-wave is an extraordinary wave. This is a well-known result of a uniaxial medium.

Further we note from (18) that the magnitudes of the z-component of the propagation vectors are different for upward and downward travelling waves, i.e., $|k_z^{\zeta u}| \neq |k_z^{\zeta d}|$. This is true in a general case. But there are situations where these magnitudes become equal. One such situation occurs when the principal axes of the medium coincide with (x, y, z), i.e., when $\psi_1 = \psi_2 = 0°$. In this limit we note that $\mathcal{A} = \mathcal{R} = 0$. Thus it is clear from (18) that z-components of the propagation vectors of the upward and downward travelling waves are equal in magnitude but opposite in sign. This result is intuitively satisfying.

We next turn our attention to the polarization vectors. We note that our $\hat{a}$ and $\hat{b}$ correspond to $\overline{D}_I$ and $\overline{D}_{II}$ in Njoku [1983][8] and Kong [1975][9]. They have denoted the characteristic waves as type I and type II waves. We further notice from (21) that for a specified propagation direction ($\hat{k}^a = \hat{k}^b$), $\hat{a}^+ \cdot \hat{b}^+ = 0$. A similar result has been reached by Kong [1975][9] and Njoku [1983][8] by a different method. Also from (21) and

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(40a) we obtain that $\hat{a}^{+} \cdot \hat{b}^{+} = 0$.

Although $\hat{a}^{+}$ and $\hat{b}^{+}$ are orthogonal, it is interesting to note that $\hat{a}^{+}$ and $\hat{b}^{+}$ are not, in general, orthogonal to each other. For we find from (39) and (40) that $\hat{a}^{+} \cdot \hat{b}^{+} = 0$ only if $\hat{k} \times \hat{o}_1 = \pm (\hat{k} \times \hat{o}_2)$. This implies that $\hat{a}^{+}$ and $\hat{b}^{+}$ are orthogonal only in the special situation where $\hat{k}$ either lies in the plane bisecting the angle between $\hat{o}_1$ and $\hat{o}_2$ or is normal to the bisecting plane.

On the other hand, in the case of a uniaxial medium the corresponding vectors are always orthogonal [Kong, 1975; Lee and Kong, 1983][9,4]. In order to clarify this apparent difference we proceed as follows. First we take the uniaxial limit of our results. In this limit we note from (45) that $(k^a)^2 = (k^o)^2$ and $(k^b)^2 = (k^e)^2$. The superscripts o an e refer to the ordinary and extraordinary waves as in Lee and Kong [1983][4]. Further (27) and (21) reduces to the following limiting values.

$$\hat{o}_1 = \hat{o}_2 = \hat{y} \sin \psi_1 + \hat{z} \cos \psi_1 \tag{46}$$

$$\hat{a}^{+} = \frac{\hat{k}^o \times \hat{o}_1}{|\hat{k}^o \times \hat{o}_1|} \tag{47a}$$

$$\hat{b}^{+} = \frac{\hat{k}^e \times (\hat{k}^e \times \hat{o}_1)}{|\hat{k}^e \times (\hat{k}^e \times \hat{o}_1)|} \tag{47b}$$

Also in this uniaxial limit

$$\hat{a}^{+} = \hat{a}^{+} \tag{48}$$

Since we have seen earlier that $\hat{a}^{+} \cdot \hat{b}^{+} = 0$, it follows that $\hat{a}^{+} \cdot \hat{b}^{+} = 0$

always in the uniaxial limit. Thus all the properties of the uniaxial medium fit logically well within the framework of the biaxial medium.

This completes the first part in our derivation of the DGF's of the two-layer biaxially anisotropic medium. In the second part [see also Mudaliar and Lee, 1991][10] we extend this work to the corresponding two-layer problem. The most important merit of this DGF lies in the repesentation. Similar representation [Lee and Kong, 1983][4] has served us well in studying and analyzing various scattering problems in uniaxially anisotropic random media [Lee and Kong, 1985; Mudaliar and Lee, 1990][11,12]. It is expected that the DGF derived here will serve similar purposes in radiation and scattering problems involving biaxially anisotropic media.

5. CONCLUSION

Due to numerous applications in various areas anisotropic medium is demanding increasing attention in recent years. Since dyadic Green's function (DGF) is a basic tool for analysis, we seek to derive the DGF for a biaxially anisotropic two-layer medium. To this end we derived in this paper the DGF for an unbounded anisotropic medium whose principal axes are arbitrarily oriented. The Fourier transform method was used to derive an auxiliary dyadic Green's function (ADGF). The DGF was then obtained by performing a linear transformation on the ADGF. Some properties of the characteristic waves, viz., the a-waves and the b-waves, were studied. Our representation of the DGF demands particular attention. Besides its compact structure it also has physical correspondence to the two characteristic waves in the medium. In the

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uniaxial limit our results agree well with those in the literature.
Extension of this work to the two-layer problem will be discussed in
another report.

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$$\mathcal{A} = 2(k_x\, \epsilon_{13} + k_y\, \epsilon_{23})/\epsilon_{33} \tag{A1}$$

$$\mathcal{R} = \left(\frac{\mathcal{A}^2}{4} - b + s\right)^{1/2} \tag{A2}$$

$$\begin{Bmatrix} \mathcal{D} \\ \mathcal{E} \end{Bmatrix} = \left[\frac{3\mathcal{A}^2}{4} - \mathcal{R}^2 - 2b \pm \frac{1}{4\mathcal{R}} \left(4\mathcal{A}b - 8c - \mathcal{A}^3 \right) \right]^{1/2} \qquad \text{if } \mathcal{R} \neq 0 \tag{A3}$$

$$\begin{Bmatrix} \mathcal{D} \\ \mathcal{E} \end{Bmatrix} = \left[\frac{3\mathcal{A}^2}{4} - 2b \pm 2\left(s^2 - 4d \right)^{1/2} \right]^{1/2} \qquad \text{if } \mathcal{R} = 0 \tag{A4}$$

where

$$b = (m_1 a_2 + c_1 + a_1 m_2 + k_x b_3 + p_{13}^2)/p_{33} \tag{A5}$$

$$c = (m_1 b_2 + b_1 m_2 + k_x c_3 + b_3 p_{13})/p_{33} \tag{A6}$$

$$d = (m_1 c_2 + c_1 m_2 + c_3 p_{13})/p_{33} \tag{A7}$$

$$m_1 = k_x k_y + p_{12} \tag{A8a}$$

$$m_2 = k_y^2 - p_{11} \tag{A8b}$$

$$p_{ij} = \omega^2 \mu\, \epsilon_{ij} \quad , \quad (i,\ j) = (1,2,3) \tag{A9}$$

$$a_1 = p_{33} - k_x^2 \tag{A10a}$$

$$b_1 = 2k_y\, p_{23} \tag{A10b}$$

$$c_1 = p_{23}^2 - (k_x^2 - p_{22})(k_\rho^2 - p_{33}) \tag{A10c}$$

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$$a_2 = k_x k_y \tag{A11a}$$

$$b_2 = k_x p_{23} + k_y p_{13} \tag{A11b}$$

$$c_2 = p_{13} p_{23} + (k_x k_y + p_{12})(k_\rho^2 - p_{23}) \tag{A11c}$$

$$b_3 = k_x(k_x^2 - p_{22}) + k_y m_1 \tag{A12a}$$
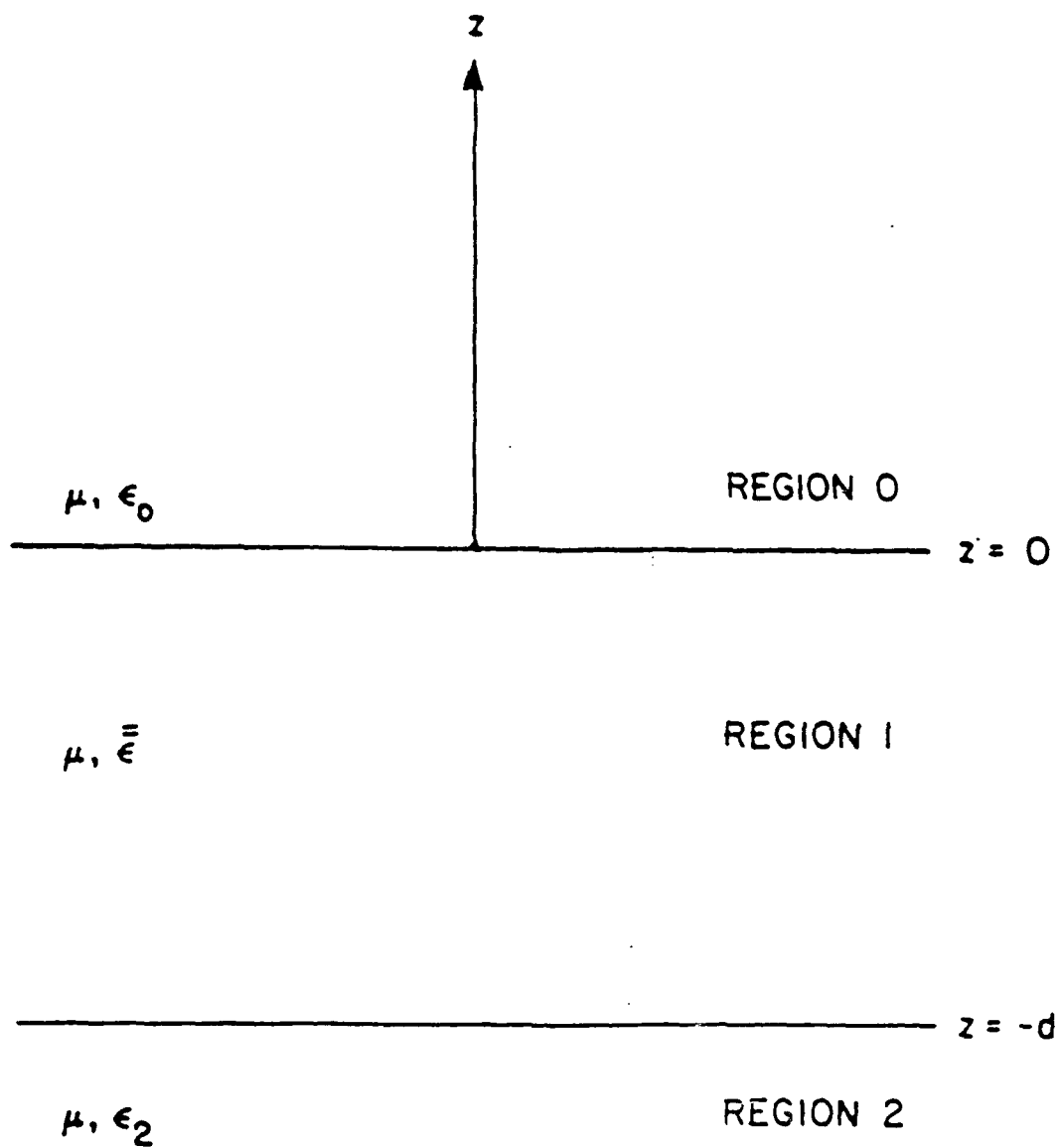
$$c_3 = m_1 p_{23} + p_{13}(k_x^2 - p_{22}) \tag{A12b}$$

$$s = \left[-\frac{B}{2} + \left(\frac{B^2}{4} + \frac{A^3}{27}\right)^{1/2}\right]^{1/3} + \left[-\frac{B}{2} - \left(\frac{B^2}{4} + \frac{A^3}{27}\right)^{1/2}\right]^{1/3} + \frac{b}{3} \tag{A13}$$

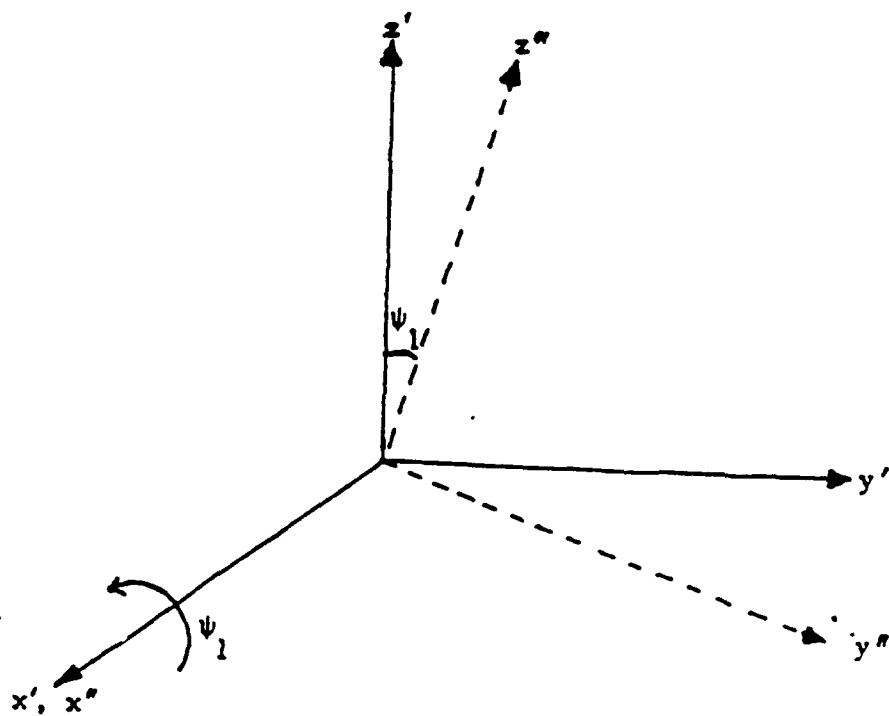$$A = \frac{1}{3}(3q - b^2) \tag{A14a}$$
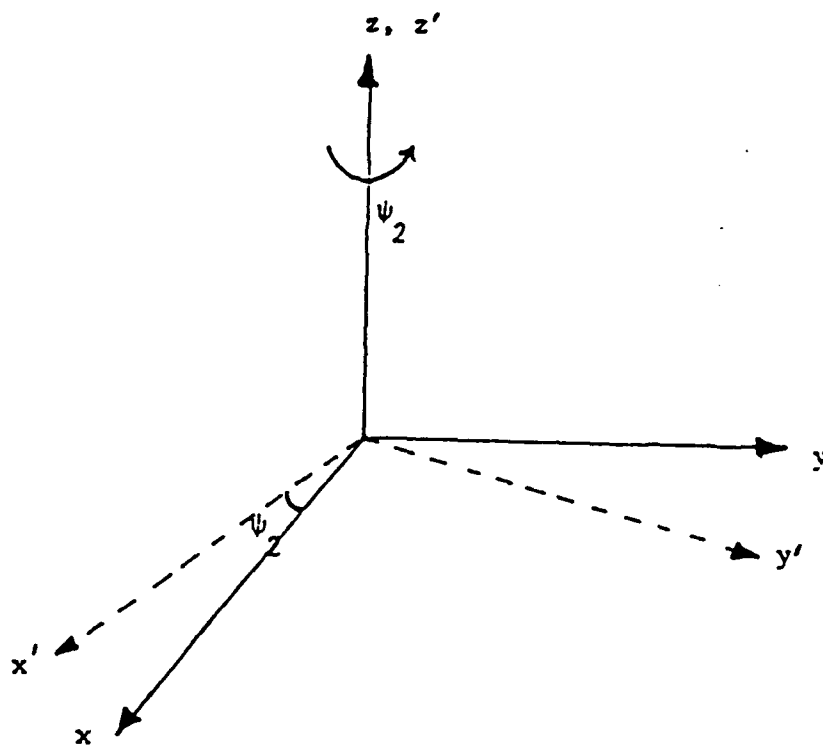
$$B = \frac{1}{27}(-2b^3 + 9bq + 27r) \tag{A14b}$$

$$q = 4c - 4d \tag{A15a}$$

$$r = -4d^2 + 4bd - c^2 \tag{A15b}$$

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z

REGION 0

μ, ϵ_0

$z = 0$

$\mu, \bar{\bar{\epsilon}}$

REGION 1

$z = -d$

μ, ϵ_2

REGION 2

(a)

(b)

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MISSION

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